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Nonlinear Systems based Prescribed-time Tracking Control for Robotic Manipulator with Time-varying Output Constraints

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Abstract: ***This paper introduces prescribed-time tracking control of nonlinear systems for robotic manipulator under time-varying output constraints (TVOC). For the robotic manipulator driven by DC motor, we will design a controller to realize: one is to realize prescribed-time tracking control; the other is no violation of the time-varying output constraints (TVOC). First, sufficient conditions for prescribed-time tracking control for nonlinear systems with TVOC are presented. Secondly, relying on the backstepping technique, the recursive design algorithm is applied to construct the controller, which then realizes the prescribed-time tracking without the violation of TVOC. Finally, the applicability of this method in robot manipulator is verified by simulation.***

Keywords: prescribed-time tracking control; robotic manipulator; time-varying output constraints.

1. INTRODUCTION

Since the field of motor control is a very broad research field, we limit the motor control to some recent nonlinear control work. Because the research of nonlinear control has made great achievements [1-6], these works have aroused people's interest in this field. For example, nonlinear control results such as feedback linearization, adaptive and robust controllers have been developed for induction motor control. For instance, in [7-8], the mechanical subsystem dynamics of a single-link direct-drive manipulator is developed.

As we all know, in real life, many systems will inevitably encounter various constraints, such as saturation, security specifications, physical shutdown and so on. Violation of these constraints may directly lead to some good performance degradation, harm or destroy the stability of the system. Therefore, dealing with the constraints in controller design has aroused extensive research interest [9-15]. In the past few years, the finite-time control with output constraints has been widely concerned, and some important results based on the Barrier Lyapunov function (BLF) to solve the output constraints have also appeared in [10-11, 16-18], for example. One of the characteristics of [16-18] is that the settling time depends on the initial conditions or system parameters and cannot be set according to our wishes.

On the other hand, the mentioned settling time is very important for finite-time control. Different methods are used to speed up the convergence and reduce the settling time. [10-11, 19-20] studied the prescribed-time stable, in which the settling time does not depend on any system parameters and initial conditions, and can be selected in advance according to our wishes. However, the results in [19-20] are valid only before the settling time. Then, the results in [10-11] are always valid, which allows the system to run continuously after the settling time has passed. In addition, the simulation results of literature [10-11] are the second-order system, while the robotic manipulator in [7-8] can be transformed into a third-order system after coordinate transformation. Therefore, is the controller designed with [10] suitable for the robotic manipulator? This problem is not involved in the existing literature, which is also part of the motivation of our current work.

Based on this, the purpose of this paper is to study the prescribed-time tracking control of robot manipulators with TVOC. For the manipulator driven by DC motor, we will design a controller to realize prescribed-time tracking with TVOC. Firstly, the sufficient conditions for the prescribed-time tracking control of nonlinear systems with TVOC are given. Secondly, based on the backstepping technology, a recursive design algorithm is used to construct the controller, so that $y(t)$ can track the desired trajectory within the settling time without violating the output constraints. Finally, the simulation results verify that the designed controller is suitable for robotic manipulator, and then verify its effectiveness.

The structure of this paper is as follows. The considered problem is described in Section 2. Section 3 provides the main content of this paper, controller design. The simulation results are shown in Section 4. Finally, section 5 gives the conclusion.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the Robotic manipulator (see Fig.1) actuated by a permanent magnet brush DC motor as in [7].

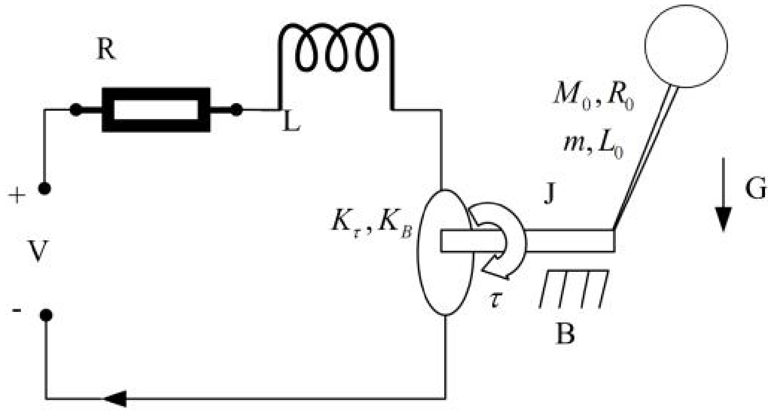

Fig.1. Robotic manipulator

The mechanical subsystem and the electrical subsystem for the permanent magnet brush DC motor are

$$M\ddot{q} + B\dot{q} + N\sin(q) = I, \quad L\dot{I} = V_e - RI - K_b\dot{q}. \tag{1}$$

2.1 Problem Formulation

The parameters have the same practical meaning as [8]. By the coordinate transformation of $x_1 = q, x_2 = \dot{q}, x_3 = I$, (1) can be further expressed as:

$$\begin{aligned} &\dot{x}_1 = x_2, \\ &\dot{x}_2 = (-N\sin x_1 - Bx_2)/M + (1/M)x_3, \\ &\dot{x}_3 = (-K_b x_2 - Rx_3)/L + (1/L)u, \\ &y = x_1, \end{aligned} \tag{2}$$

where x_1 is the position of the load, x_2 is the velocity of the load, x_3 is the motor armature current, u is the input control voltage, R is the armature resistance, L is the armature inductance, K_b is the back-emf coefficient, and N , M and B are constants given by

$$\begin{aligned} &M = (J + mL_0^3/3 + M_0L_0^2 + 2M_0R_0^2/5)/K_\tau, \\ &N = (mL_0g/2 + M_0L_0g)/K_\tau, \\ &B = B_0/K_\tau, \end{aligned} \tag{3}$$

in which J is the rotor inertia, m is the link mass, L_0 is the link length, M_0 is the load mass, R_0 is the radius of the load, B_0 is the coefficient of viscous friction at the joint, g is the gravitational acceleration, and K_τ the coefficient which characterizes the electromechanical conversion of armature current to torque.

2.2 Time-varying output constraints

The asymmetric time-varying output constraint (TVOC) can be described as follows:

$$\underline{k}_{c_1}(t) < y(t) < \overline{k}_{c_1}(t), \forall t \geq 0, \tag{4}$$

where $\overline{k}_{c_1} : R_+ \to R$ and $\underline{k}_{c_1} : R_+ \to R$ are continuous functions with $\overline{k}_{c_1}(t) > \underline{k}_{c_1}(t), \forall t \geq 0$.

According to [4], we requires the asymmetric time-varying output constraints $\overline{k}_{c_1}(t)$ and $\underline{k}_{c_1}(t)$ to be bounded, whose derivatives $\overline{k}_{c_1}^{(i)}(t)$ and $\underline{k}_{c_1}^{(i)}(t)$, $i=1,\cdots,n,$ are also bounded.

2.3 Control objectives

In this paper, we will deal with the problem of the prescribed-time tracking with TVOC (4), that is, for the permanent magnet brush DC motor (2), we will design a controller to realize the following characteristics:

(i) $y(t)$ prescribed-time tracks the desired trajectory $y_d(t)$ within settling time t_f, where t_f is independent of any system parameters and initial conditions;

(ii) the TVOC (4) is not violated;

3. CONTROLLER DESIGN

In this section, our main purpose is to design the controller to ensure prescribed-time tracking control while the TVOC is not violated.

Step1.

First, define the output tracking error variable:

$$z_1 = x_1 - y_d. \tag{5}$$

Choose the asymmetric time-varying BLF as [3]:

$$V_1 = \frac{q(z_1)}{2p}\ln\frac{k_{b_1}^{2p}(t)}{k_{b_1}^{2p}(t)-z_1^{2p}} + \frac{1-q(z_1)}{2p}\ln\frac{k_{a_1}^{2p}(t)}{k_{a_1}^{2p}(t)-z_1^{2p}}, \tag{6}$$

where $p \in Z_+$, the function $q(\cdot)$ is defined as

$$q(\cdot) := \begin{cases} 1, & \text{if } \cdot > 0; \\ 0, & \text{if } \cdot \le 0, \end{cases} \tag{7}$$

and

$$k_{a_1}(t) := y_d(t) - \underline{k}_{c_1}(t), \tag{8}$$

$$k_{b_1}(t) := \overline{k}_{c_1}(t) - y_d(t). \tag{9}$$

The stabilizing function $\alpha_i, i=1,2,3$ are C^{3-i}. We change the error coordinates (5) by

$$\xi_a = \frac{z_1}{k_{a_1}}, \quad \xi_b = \frac{z_1}{k_{b_1}}, \quad \xi = q\xi_b + (1-q)\xi_a. \tag{10}$$

Then, (6) is written as

$$V_1 = \frac{1}{2p}\ln\frac{1}{1-\xi^{2p}}, \tag{11}$$

we know that V_1 is positive definite and continuously differentiable when $|\xi|<1$.

Derivation of (5) with respect to time t, one has

$$\dot{z}_1 = x_2 - \dot{y}_d. \tag{12}$$

Define

$$h(t) := \begin{cases} 1, & \text{if } 0 \le t < t_f; \\ 0, & \text{if } t \ge t_f. \end{cases} \tag{13}$$

For simplicity, we use h instead of $h(t)$. Then, the stabilization function α_1 can be designed as:

$$\alpha_1 = h(\phi_1 - \psi z_1) + (1-h)\phi_1, \tag{14}$$

where $\psi = \dfrac{\eta}{t_f - t}$ and $\phi_1 = -(\kappa_1 - \kappa(t))z_1 + \dot{y}_d$ with

$$\kappa(t) = \sqrt{\frac{\dot{k}_{a_1}^2(t)}{k_{a_1}^2(t)} + \frac{\dot{k}_{b_1}^2(t)}{k_{b_1}^2(t)} + \beta} := \sqrt{K(t) + \beta} \tag{15}$$

for $\kappa_1, \beta \in Z_+$ and $\eta > np$. We get that $\kappa(t)$ and $\dot{\kappa}(t)$ are bounded. Here, the term ψz_1 is called as an extra fractional term.

Then, the subsystem (12) becomes

$$\dot{z}_1 = h[-(\kappa_1 + \kappa(t))z_1 + z_2 - \psi z_1] + (1-h)[-(\kappa_1 + \kappa(t))z_1 + z_2], \tag{16}$$

where the error variable is

$$z_2 = x_2 - \alpha_1. \tag{17}$$

Differentiating V_1 along the trajectory of (16) yields

$$\dot{V}_1 = \frac{q\xi_b^{2p-1}}{k_{b_1}(t)(1-\xi_b^{2p})}(\dot{z}_1 - z_1\frac{\dot{k}_{b_1}(t)}{k_{b_1}(t)}) + \frac{(1-q)\xi_a^{2p-1}}{k_{a_1}(t)(1-\xi_a^{2p})}(\dot{z}_1 - z_1\frac{\dot{k}_{a_1}(t)}{k_{a_1}(t)}). \tag{18}$$

Due to $\ln\dfrac{1}{1-\xi^{2p}} < \dfrac{\xi^{2p}}{1-\xi^{2p}}$ and $\kappa(t) + q\dfrac{\dot{k}_{b_1}(t)}{k_{b_1}(t)} + (1-q)\dfrac{\dot{k}_{a_1}(t)}{k_{a_1}(t)} > 0$, we obtain that

$$\begin{aligned} \dot{V}_1 &\le -\frac{\xi^{2p}}{1-\xi^{2p}}(\kappa_1 + \psi) + \mu z_1^{2p-1} z_2 \\ &\le -2pV_1(\kappa_1 + \psi) + \mu z_1^{2p-1} z_2 \\ &\le -2pV_1\psi + \mu z_1^{2p-1} z_2, \end{aligned} \tag{19}$$

where $\mu := q/(k_{b_1}^{2p}(t) - z_1^{2p}) + (1-q)/(k_{a_1}^{2p}(t) - z_1^{2p})$.

Step2.

Derivation of (17) with respect to time t, we have

$$\dot{z}_2 = (-N\sin x_1 - Bx_2)/M + (1/M)x_3 - \dot{\alpha}_1, \tag{20}$$

The stabilizing function α_2 can be designed as:

$$\alpha_2 = Mh(\phi_2 - \psi z_2) + M(1-h)\phi_2, \tag{21}$$

where $\phi_2 = (N\sin x_1 + Bx_2)/M - \kappa_2 z_2 - \mu z_1^{2p-1} + \dot{\alpha}_1$ with $\kappa_2 \in Z_+$.

From (21) and (20), one has

$$\dot{z}_2 = h[-\mu z_1^{2p-1} - \kappa_2 z_2 + \frac{1}{M}z_3 - \psi z_2] + (1-h)[-\mu z_1^{2p-1} - \kappa_2 z_2 + \frac{1}{M}z_3], \tag{22}$$

where the error variable is

$$z_3 = x_3 - \alpha_2. \tag{23}$$

Define the function

$$V_2 = V_1 + \frac{1}{2}z_2^2. \tag{24}$$

Then, when $0 \le t < t_f$, one has

$$\begin{aligned}\dot{V}_2 &\le -2pV_1\psi + \mu z_1^{2p-1} z_2 + z_2(-\mu z_1^{2p-1} - \kappa_2 z_2 + \frac{1}{M} z_3 - \psi z_2) \\ &\le -2pV_1\psi - \psi z_2^2 - \kappa_2 z_2^2 + \frac{1}{M} z_2 z_3.\end{aligned} \tag{25}$$

Step3.

Derivation of (23) with respect to time t, we get

$$\dot{z}_3 = (-K_b x_2 - Rx_3)/L + (1/L)u - \dot{\alpha}_2, \tag{26}$$

Design the following stabilizing function α_3:

$$u = \alpha_3 = Lh(\phi_3 - \psi z_3) + L(1-h)\phi_3, \tag{27}$$

where $\phi_3 = (K_b x_2 + Rx_3)/L - \kappa_3 z_3 - \frac{1}{M} z_2 + \dot{\alpha}_2$ with κ_3 is positive constant.

Substituting (27) into (26) yields

$$\dot{z}_3 = h[-\frac{1}{M} z_2 - \kappa_3 z_3 - \psi z_3] + (1-h)[-\frac{1}{M} z_2 - \kappa_3 z_3]. \tag{28}$$

Define the function

$$V_3 = V_2 + \frac{1}{2} z_3^2. \tag{29}$$

When $0 \le t < t_f$, we get

$$\begin{aligned}\dot{V}_3 &\le -2pV_1\psi - \psi z_2^2 - \kappa_2 z_2^2 + \frac{1}{M} z_2 z_3 + z_3(-\frac{1}{M} z_2 - \kappa_3 z_3 - \psi z_3) \\ &= -2pV_1\psi - \psi \sum_{i=2}^{3} z_i^2 - \sum_{i=2}^{3} \kappa_i z_i^2.\end{aligned} \tag{30}$$

Theorem 1. The closed-loop system has the following properties:

(i) $z_1(t)$ converges to zero in the settling time t_f.

(ii) The stabilizing function $\alpha_i, i = 1,2,3$ are C^{3-i}.

(iii) The out constraint $\underline{k}_{c_1}(t) < y(t) < \overline{k}_{c_1}(t)$ is held for $t \ge 0$.

(iv) $z_1 \equiv 0, \forall t \ge t_f$.

The proof of this theorem again relies on the backstepping method. Following the same method as in the proof of Theorem 1 [10], we have omitted it for simplicity.

4. SIMULATION RESULTS

To demonstrate the effectiveness of the designed controller, we will apply the proposed method to the permanent magnet brush DC motor (2). For simulations, set the following parameters that are the same as those in [7]:

$J = 1.625 \times 10^{-3} Kg\ m^2$, $m = 0.506 Kg$, $M_0 = 0.434\ Kg$, $L_0 = 0.305\ m$, $R_0 = 0.023\ m$,

$B_0 = 16.25 \times 16^{-3} N\ m\ s / rad$, $L = 25.0 \times 10^{-3}\ H$, $R = 5.0\ \Omega$, $K_\tau = K_b = 0.90\ N\ m / A$,

$and\ g = 9.8\ m / s^2$.

For the permanent magnet brush DC motor (2), let the required reference trajectory be $y_d(t) = -0.2\cos t + 0.2$ and the asymmetric TVOC be $\underline{k}_{c_1}(t) = -0.7 + 0.4\cos t$ and $\overline{k}_{c_1}(t) = 0.8 + 0.1\sin t$.

For the initial state $x(0) = (-0.1, 3, 0.1)^T$ and the different settling times $t_f = 5$ and $t_f = 15$. For simulations, let $\kappa_1 = 2$, $\kappa_2 = 0.1$, $\kappa_3 = 1$, $\beta = 1$ and $\eta = 2.1$.

Firstly, Fig. 2 show the tracking error trajectory $z_1(t)$, where $z_1(t)$ is always between $-k_{a_1}(t)$ and $k_{b_1}(t)$, and converges to zero within different settling times t_f.

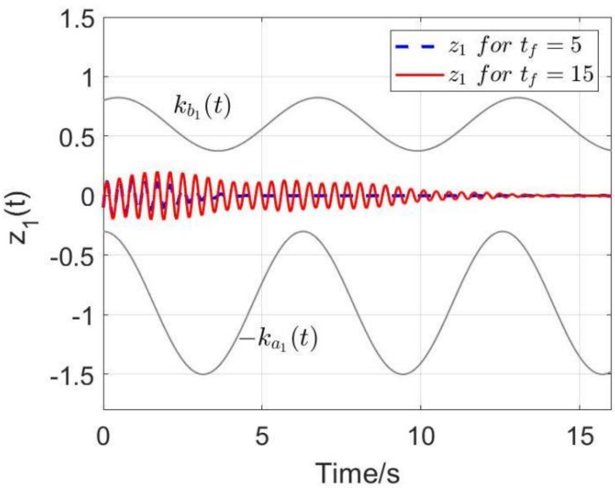

Fig.2. Tracking error trajectory

Then, from Fig.3, we can see that when realizing asymmetric TVOC, $y(t)$ prescribed-time converge to $y_d(t)$ within different settling times t_f.

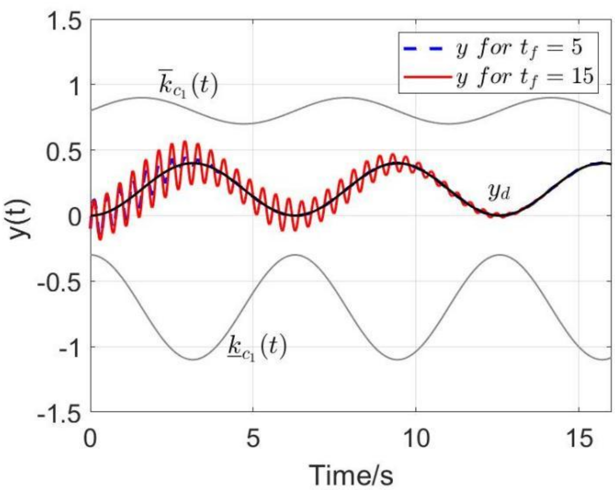

Fig.3. Output tracking trajectory

Finally, for $t_f = 1$, the magnitude of controller is plotted in Fig.4.

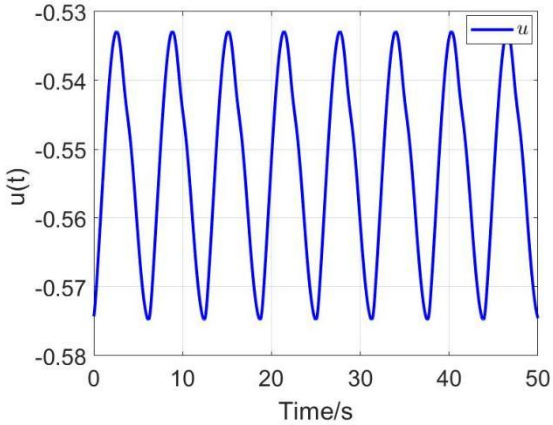

Fig.4. The magnitude of the controller

5. CONCLUSIONS

Prescribed-time tracking control for robotic manipulator under time-varying output constraints is addressed in this paper. For the robotic manipulator, the finite-time controller is proposed to tackle with the problem that the tracking error converges to zero within any desired settling time and remains zero thereafter without the violation of the output constraint. In addition, by applying the recursive design algorithm, we construct a stabilizing function to decreases the asymmetric time-varying BLF to the origin, And without the violation of the TVOC. Finally, the simulation verifies the effectiveness of the proposed control strategy.

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