

Mathematical Strategies from the Perspective of Multi representation Variant Teaching – Taking Calculus in Senior High School as an Example

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Abstract: *This paper focuses on the integration of multiple representations and variable teaching into calculus in senior high school. First, it analyzes the difficulties faced by calculus learning, then understands the thinking and skills about calculus, and finally designs the teaching process of calculus in senior high school based on the perspective of multiple representations and variable teaching. The integration of multiple representations and variant teaching aims to develop students' core competencies in mathematics (the core competencies mentioned below do not specifically refer to the core competencies in mathematics), enabling them to acquire lifelong learning abilities. This article advocates starting from the perspective of knowledge itself and students, and hopes to provide some strategies for teachers' teaching.*

Keywords: Multiple representations; Variant teaching; Calculus.

1. INTRODUCTION

Mathematics learning lies in a holistic and detailed understanding of knowledge. Variant teaching is beneficial for students to grasp the essence of knowledge and assimilate new knowledge. Multiple representations tend to understand the origin of knowledge, and different representations can lead to different ways of thinking for students. The two promote each other and are conducive to the common development of students' core literacy. Hawthorn slices, also known as hawthorn cakes, are made of high-quality hawthorn and are a favorite food. After eating, there is the effect of Shengjin appetizer, digestive, blood pressure lowering, digestion and digestion, and promoting blood circulation. The traditional hawthorn piece manufacturing process is divided into multiple processes such as material selection, cleaning, steaming, beating, forming, drying and packaging. The traditional hand-made hawthorn piece manufacturing process can no longer satisfy the market demand of Hawthorn Flakes. Most of the steps of making Hawthorn Flakes have been able to be produced by machinery through several generations of research and development, and the production of Hawthorn Flakes has become more mechanized and automated. It has greatly improved the production efficiency of Hawthorn tablets and provided a guarantee for the market supply of Hawthorn tablets.

The outer layer is the brand of raw hawthorn slices, raw materials, production date, shelf life, manufacturer. Wait. It takes time and effort to package the hawthorn slices by hand, and the efficiency is not guaranteed. At present, the hawthorn packaging machine on the market in China is expensive, and the price is more than 100,000. It is difficult for small and medium-sized manufacturers to purchase. Therefore, it is particularly necessary to make a machine that is low in cost and can replace the manual packaging of hawthorn slices. The hawthorn packaging machine designed by us uses the motor as the power to drive the biasing of the crank slider mechanism through the chain drive to realize the mechanical reciprocating motion, thereby completing the rapid packaging of the hawthorn cake. The mechanical device has the advantages of simple structure, durability, high efficiency and low cost. It is an economical packaging machine.

1.1 Difficulties in learning calculus

Calculus is a part of the content that students have not been exposed to in junior high school. Some symbols and concepts about calculus are relatively abstract. Students are accustomed to intuitive knowledge, and they inevitably have no way to deal with abstract knowledge. They do not know how to understand it, and suddenly change from constant to variable thinking mode. The span of knowledge structure is relatively large, and there are

many variations of topics, which is difficult for students to grasp. When introducing the concept of derivative, students are unable to integrate the content in class with actual life, and are unable to transfer previously learned knowledge to new knowledge, which reduces students' enthusiasm for learning. In addition, the teacher's unclear understanding of students' existing knowledge framework leads to poor teaching effectiveness.

1.2 Carrier of calculus Thought

The carrier of calculus thought can be summarized as operation, image and model. Operation can help students to establish number sense and improve logical reasoning ability. In other words, logical reasoning ability is also the basis for students to develop mathematical computing ability; Image is an important way for teachers to guide students to learn derivatives, and it is also a form of multivariate representation learning. Many contents in calculus involve functions, and the application of functions and images are also inextricably linked. In the application of derivatives, multiple representations can help students construct models and understand conceptual meanings, which is conducive to their comprehensive understanding of knowledge. Students need to solve practical problems by constructing mathematical models, and they can construct different mathematical models to express different life situations.

2. TEACHING DESIGN OF CALCULUS IN SENIOR HIGH SCHOOL FROM THE PERSPECTIVE OF MULTIVARIATE REPRESENTATION VARIANT TEACHING

The Hawthorn cake packing machine is mainly used in the automatic packaging of the Hawthorn cake (small money) in the Hawthorn food factory. This work has completed the design of the packaging structure and sealing structure of the Hawthorn pie packing machine. The packaging structure and sealing structure of the Hawthorn packing machine are powered by two motors. All of them use 50Hz, 220V AC power supply and chain drive. Finally, the automatic package of the Hawthorn pie packing machine is realized. The function of loading. This work is simple in structure, easy to operate, low cost, can effectively solve the Hawthorn slices in the production of packaging and transportation problems, improve work efficiency. Under the dynamic load, the metal will gradually accumulate damage, crack and expand. After a finite cycle, it will eventually destroy, this phenomenon is called fatigue. Bridge steel structural members of steel bridges also have this kind of fatigue property. During the service process, the bridge carries the dynamic load of the vehicle, the changing crowd load and the wind load day after day, the steel bridge has the variable amplitude alternating stress effect on the rigid structure of each part, which will cause local permanent cumulative damage of the steel bridge, resulting in small fatigue cracks, with the continuous expansion of cyclic stress action, cracks will form, resulting in structural deterioration, thus affecting the safe use of the bridge.

Once the fatigue fracture of bridge steel occurs, it will often lead to serious accidents of bridges, causing heavy personnel and economic losses. On December 15, 1967, the Silver Bridge on the Ohio River collapsed completely in a short period of time, killing 46 people. The investigation of the debris pointed out that it was the fatigue fracture of a steel member on the bridge that caused the accident. On October 21, 1994, the Seongsu Bridge in Seoul, south Korea suddenly collapsed. Six vehicles, including a bus full of students and a car full of police, crashed into the Han River, killing 32 people and injuring 17 others. The investigation found that one of the reasons for the collapse was the fatigue of a section of metal in the middle of the bridge. With the continuous development of China's infrastructure construction, the traffic volume increases rapidly and the traffic load has become heavier. The fatigue problem of bridge steel is also more important and needs to be paid enough attention.

2.1 Construction of calculus Learning Model Based on Multiple Representation Variant Teaching

Multi representation is a teaching method advocated in the West, which tends to be based on multi learning. Variant teaching is a traditional teaching method in China that leans towards essential learning. Variant teaching based on multi representation is beneficial for students to grasp the knowledge structure as a whole, diverge their thinking, and cultivate their innovative ability on the basis of their understanding of the essence of knowledge. Representation can include symbolic representation, verbal representation, and graphical representation, which is the representation of knowledge. The external form and internal essence are interrelated, and each learner's field of expertise is different. Some are good at intuitive imagination, while others are good at verbal expression. In

learning materials, each person can improve the meaning of learning and develop students' core literacy through their own good representation form. Through communication between students, they can strengthen their diverse understanding of knowledge. Variant teaching can strengthen students' understanding of the essence of knowledge by changing some forms of extension, such as introducing the relationship between derivatives and tangents through previously learned functions, correcting incorrect guesses with "giving counterexamples", and helping students understand the essence of knowledge with "one question changing". The teaching process of calculus should take "problems or situations introduced by teachers" as the starting point, multiple representations as the carrier, variable teaching as the means, forming calculus thinking as the goal, and solving problems as the end result. The specific process of variable teaching based on multiple representations is as follows: posing problems (life situations, existing experience) → multiple representations → variable problems (Problem from simple to difficult) → Problem solving (pointing to thinking quality and emotional attitude values) Abstract representation (the essence of knowledge, equivalent substitution). Taking the formation of the concept of derivative as an example, variations are made based on verbal representation: the instantaneous speed is derived from the average speed of the athlete, and the instantaneous rate of change from the average rate of change of the line segment to the point; Based on symbolic representation Variant.

2.2 Teaching Design

(1) Pre class preparation

Evaluate literacy situation, subdivide knowledge goals and literacy goals, and start teaching with problem driven approach, cultivating students' ability to think with problems and allowing them to explore in small groups. Is the concept of derivative a process or a result? By comparing the average rate of change, you can try using images To represent the instantaneous rate of change? What is the relationship between non definite integral and definite integral? Is there a second way to solve definite integral? Regardless of whether students answer correctly or not, teachers should provide motivational evaluations aimed at encouraging students to think independently and develop core competencies such as mathematical modeling and abstraction.

(2) In class strategies

Creating a reasonable context: Before introducing extreme thinking, teachers can first introduce examples from daily life. For example, the display screen of a TV utilizes the principle of extreme thinking. More than a decade ago, when the signal on the TV was poor, many gray and white small blocks would appear. In fact, the displayed image is inherently serrated, but due to its high density, what appears in front of us is a smooth and clear image. The same applies to pixels, which themselves are composed of many small squares, each with its own clear color and position, which directly affects what the image we see looks like. By connecting with real life and enhancing students' interest in learning, classroom effectiveness can be achieved with half the effort.

Try multiple representations: To help students accurately understand the concept of derivatives, they can use their internalization of old knowledge and promote the cultivation of students' core literacy through the mutual transformation of various representations. When students first came into contact with function problems in junior high school, they first introduced linear function with the combination of tables and images, followed by quadratic function, which gave students a preliminary understanding of the relationship between multiple representations. By high school calculus, part of the content was still closely related to function problems, and many topics required students to analyze derivative functions, When encountering complex functions, teachers should guide students to try multiple representations (such as image representation, symbol representation, speech representation), and use the idea of combining numbers and shapes to find suitable mathematical methods. Take the concept of definite integral as an example:

1) Verbal representation

The first step is to divide the area of the curve, the second step utilizes the idea of replacing the curve with a straight line, the third step is to sum the area of each small curved trapezoid, and the fourth step is to take the limit. When we divide into n trapezoidal surfaces, the larger n , the smaller the error. When $n \rightarrow \infty$, the error is the smallest.

2) Graphical representation

First, draw a function image about $y=x^2$, leaving only the right half. Then, draw the line $x=1$, calculate the area between them, and divide them into small curved trapezoids for students to understand. The teacher shows three different sizes of segmentation, and students can easily see that the finer the segmentation, the smaller the error. (Graphic omitted)

3) Symbolic representation

The ability of image and logical reasoning has deepened the students' understanding and application of definite integral, realizing the transformation from "learning" to "learning".

Design of variable teaching: the core of variable teaching is "one problem is changeable". The constant abstraction of mathematical problems aims to stimulate students' learning motivation, enable students to view problems with critical thinking. Variable problems progress from shallow to deep levels, enhance students' confidence in learning mathematics, achieve from shallow learning to deep learning, and further cultivate students' mathematical abstraction. When learning calculus, many students think that extreme value is the maximum value. The essence is that they do not understand that there can be many increase and decrease intervals of complex functions. When transforming knowledge, teachers can guide students to extend their already learned knowledge and obtain new conclusions that can be used as new theorems, cultivating students' logical reasoning and mathematical abstraction skills. Taking the use of derivatives to solve tangent equations as an example:

Example 1: Find the tangent equation of the function $y=f(x)=x^2$ at $x_0=2$. Example 2: Solve the tangent equation of function $y=x^2$ passing through points (3, 5).

Example 3: The tangent equation of $y=x^2$ at point p is parallel to the straight line $4x+y+2=0$, find P Coordinate and tangent equation of points.

From the three example questions, we can see that the difficulty of the questions increases from easy to difficult, which is in line with students' cognitive rules. Example 1 is a simple application of using derivatives to solve tangent equations. Example 2 solves problems where points are not on the tangent line, which combines the idea of equivalent substitution with new knowledge to solve mathematical problems. Example 3 requires students to find the coordinates of point p through equal slopes, providing ideas for solving more complex problems. Example 1 is directly related to students' cognition and can also be understood. For shallow learning, Example 2 and Example 3 are variations based on Example 1, requiring students to combine new and old knowledge, which is a cognitive leap. The transformation process of a question can supplement and change some conditions, requiring the problem-solver to exert intuitive imagination and involve various thinking methods such as association and analogy. Through such a series of transformations, students' intuitive imagination ability is also cultivated.

Using information technology: The use of multimedia technology is beneficial for students to intuitively see the extensive connections between multiple representations, and the variations and image changes of the questions are also more intuitive, increasing students' interest in learning mathematics while cultivating their ability to visualize. However, in the process of variant teaching, it is recommended that teachers use screens to display the changes in the questions and write the problem-solving process with chalk.

(3) Follow up after class

Design thinking: the teacher tries to let students design a mind map about the content of this section to guide students to review and reflect on the representations and problem strings mentioned in the class. The connection between knowledge and knowledge can be a problem-solving approach or a perspective of multiple representations: Then ask the students to think, what is the relationship between the curved area and n ? Is it a split The more trapezoidal surfaces are cut, the smaller the error? So when the above formula is $n \rightarrow \infty$, the closer we get to the curved area, and when $n \rightarrow \infty$, the result we get is 1.

3. THE ABOVE INTRODUCES THREE TYPES OF REPRESENTATIONS, ENRICHING THEIR FORMS AS MUCH AS POSSIBLE.

The combination of multiple representations is beneficial for improving students' ability to recognize mathematical thinking methods. It provides direction for some students in a confused state, deepens their understanding of the mathematical meaning behind symbols, and attempts to abstract mathematical concepts from different angles. From the perspective of core competencies, a single logical thinking system is not conducive to the development of students' logical thinking. Multiple representations summarize commonalities in various ways to form concepts, making students' thinking training more comprehensive and reasonable, and improving students' mathematical skills.

The concept of knowledge can be a variant question about the same knowledge point, aimed at deepening students' understanding deep understanding of knowledge. Attempting autonomous propositions: Here, teachers need to guide students in the direction of autonomous propositions, allowing them to try multiple solutions to one problem, thinking about whether there are other representations besides the ones introduced in class, whether making some changes to the problem (such as adding, changing, or deleting conditions) can generate new mathematical problems, and whether further extensions can be made to continue with the variations from the problems introduced in class? This process mainly cultivates students' ability to think independently, constructs their own representation variations of mathematical concepts and relationships, recognizes that the knowledge content of calculus is the abstraction of specific models, and further develops logical reasoning, intuitive imagination and mathematical abstraction literacy. Conducting Mathematics Activities: After the learning of a chapter is completed, the teacher can conduct an activity class. The main content of the course is to construct real-life scenarios in groups, perfectly integrate mathematics with real-life situations, give each student a different identity, enhance the connection between campus learning and society, and enable students to experience the process of knowledge development in practice. In variant inquiry teaching, teachers can use variant exercises to help students find experience in incorrect understandings, share failed experiences, and cultivate students' divergent thinking. In this process, teachers should pay attention to not only revealing the practical background of knowledge, but also emphasizing the application of knowledge, establishing a knowledge community for students, and creating a collaborative and competitive learning environment, Guide students to form correct values and outlook on life. After the activity, provide students with objective evaluation, and combine self-evaluation with self-evaluation to promote students' comprehensive development and progress.

3.1 Effect of Stress Concentration

Fatigue is very sensitive to defects such as notches, weld defects and cracks. Stress concentration at the defect location accelerates the initiation and expansion of fatigue failure. In the welding part, due to the limitation of the welding process, it is easy to produce welding defects such as pores, incomplete fusion, slag inclusions or hot and cold cracks. These defects provide a large-scale crack source for fatigue, while stress concentration occurs, making the crack spread faster. As the crack develops gradually, the effective bearing area of the section decreases correspondingly, and the stress concentration phenomenon is also more serious, which further promotes the continued expansion of the crack.

Among the various weld defects of steel on the fatigue strength, the effect of the non-weld weld is the largest, followed by the slag, and the smallest is the void. The length of the steel slag inclusions has a great influence on the fatigue strength, but the number of inclusions is small.

The type of connection determines the geometry of the joint, so the different types of connections have different levels of stress concentration. The butt joint has a small stress concentration factor and its fatigue strength is higher than other joint forms. But there are also special cases, such as butt joints with permanent gaskets, which have much lower fatigue strength than ordinary butt joints because of the severe stress concentration at the gasket. T-joints and cross-links are widely used in welded steel structures, but due to the obvious change in cross-section of the transition section, the stress concentration of the two welds is greater than that of the butt joint, and the fatigue strength is lower than that of the butt joint.

3.2 Effect of Stress Amplitude

One of the main factors controlling fatigue is the stress amplitude. In the process of smelting, rolling, welding, hot-shaping, etc., due to the uneven thermal process, the residual tensile stress of the value close to (or equal to) the yield point will be generated in the weld zone. They are produced during the welding shrinkage process and change their actual stress state. This residual tensile stress is always involved in the cyclic stress process, superimposed or offset with the stress generated by the external load, which shifts the actual maximum or minimum stress. But no matter what form of cyclic stress spectrum, the difference between the maximum stress and the minimum stress can be used to express the stress amplitude. It can be seen that the fatigue strength of the welded structure is independent of the nominal maximum stress and is related to the stress amplitude.

The stress amplitude is usually used as the stress index of the fatigue resistance of the welded structure. By comparing the critical stress amplitude caused by fatigue failure under a certain number of cycles with the corresponding allowable stress amplitude, it can be judged whether the fatigue resistance requirement is met. For variable amplitude stress, equivalent conversion is required.

3.3 Effect of Cycle Times

Under different stress amplitudes, the number of stress cycles caused by fatigue failure of various components and connections is different. The greater the stress amplitude, the less the number of cycles, otherwise the more. When the stress amplitude is less than a certain value, fatigue failure will not occur even after an infinite number of cycles. The fatigue life of steel is the number of stress cycles experienced before fatigue failure. In order to get better fatigue strength, we can start from two aspects to improve the fatigue life. On the one hand, try to use steel with high finish and smooth transition in appearance. On the other hand, try to eliminate tensile stress and preset pressure stress. Hawker's technology perfectly combines these two aspects, while meeting the requirements of these two aspects, greatly improving the fatigue life.

4. CONCLUSION AND OUTLOOK

From the perspective of integration of multiple representation and variant teaching, this paper designs the overall teaching idea of calculus through the multiple representation of "examples" and variant teaching, aiming to help students establish a complete knowledge structure framework, activate their original mathematical experience, help students cultivate innovative thinking on the basis of deepening their understanding of the nature of knowledge, and develop their ability to think independently and learn for life, so that calculus thinking can be well applied in problem solving.

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